J Geod (2008) 82:389–399
DOI 10.1007/s00190-007-0187-4

ORIGINAL ARTICLE

Resolution of GPS carrier-phase ambiguities in Precise Point Positioning (PPP) with daily observations

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Received: 24 November 2006 / Accepted: 13 August 2007 / Published online: 23 October 2007

Abstract Precise Point Positioning (PPP) has been demonstrated to be a powerful tool in geodetic and geodynamic applications. Although its accuracy is almost comparable with network solutions, the east component of the PPP results is still to be improved by integer ambiguity fixing, which is, up to now, prevented by the presence of the uncalibrated phase delays (UPD) originating in the receivers and satellites. In this paper, it is shown that UPDs are rather stable in time and space, and can be estimated with high accuracy and reliability through a statistical analysis of the ambiguities estimated from a reference network. An approach is implemented to estimate the fractional parts of the single-difference (SD) UPDs between satellites in wide- and narrow-lane from a global reference network. By applying the obtained SD-UPDs as corrections to the SD-ambiguities at a single station, the corrected SD-ambiguities have a naturally integer feature and can therefore be fixed to integer values as usually done for the double-difference ones in the network mode. With data collected at 450 stations of the International GNSS Service (IGS) through days 106 to 119 in 2006, the efficiency of the presented ambiguity-fixing strategy is validated using IGS Final products. On average, more than 80% of the independent ambiguities could be fixed reliably, which leads to an improvement of about 27% in the repeatability and 30% in the agreement with the IGS weekly solutions for the east component of station coordinates, compared with the real-valued solutions.

Keywords GNSS · Precise Point Positioning (PPP) · Uncalibrated phase delay · Integer ambiguity resolution · Global network

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1 Introduction

While network analysis strategies for deriving integrated estimation of station coordinates and satellite orbits as well as Earth rotation parameters with full statistical information are steadily improved (Ge et al. 2006b), Precise Point Positioning (PPP) (Zumberge et al. 1997; Kouba and Heroux 2001) becomes a very pragmatic tool that reduces the computation burden for applications where co-variances among parameters of different stations are not of interest. PPP is not only widely used in crustal deformation monitoring (Azuá et al. 2002; Savage et al. 2004; Hammond and Thatcher 2005; D'Agostino et al. 2005; Calais et al. 2006), near real-time GPS meteorology (Gendt et al. 2003; Rocken et al. 2005) and orbit determination of low Earth orbiting satellites (Bock et al. 2003; Zhu et al. 2004), but is also applied in the precise positioning of mobile objects (Gao and Shen 2002; Zhang and Andersen 2006). It becomes even more important as more and more dense networks are deployed for regional seismic activity and meteorological monitoring.

However, the PPP accuracy in the station east component, measured by its repeatability, is not as good as that of the network solutions (Calais et al. 2006), although their results are comparable as shown in most of the above-mentioned applications. It is well known that in the network mode, the east component can be improved significantly by resolving the integer carrier-phase ambiguities. Such fixing is even more important for kinematic applications or static positioning with short-time observations, where results are improved dramatically. In expecting a similar improvement for PPP, its ambiguity fixing is considered as one of the innovative issues for GNSS (Global Navigation Satellite System) research and applications in the next ten years (Rizos 2006).

The major problem for the PPP ambiguity fixing is that the zero-difference (ZD) ambiguity of a satellite-receiver pair or the single-difference (SD) ambiguity between two satellites is naturally not an integer value, due to the existence of the uncalibrated phase delays (UPD) originating in the receiver and satellite (Blewitt 1989). Until now, only double-difference (DD) ambiguities can be fixed, because there the UPDs cancel. By combining PPP solutions of simultaneously observed stations, DD-ambiguities can be defined and fixed in the same way as for network solutions (Zumberge et al. 1997; Blewitt 2005). However, it brings back the problem of the computational burden into the data analysis, which is often solved by fixing ambiguities in sub-network mode (Savage et al. 2004; Hammond and Thatcher 2005; D'Agostino et al. 2005).

The fact that in global networks more than 97% of the DD-ambiguities can be resolved to integer values after optimizing their selection and that the fixing rate is very slightly correlated with the baseline length (Ge et al. 2005) implies that the fractional parts of the two SD-ambiguities, which form the DD-ambiguity, must agree with each other very well. Otherwise, their difference would not be close to an integer. In other words, the SD-ambiguities of a certain satellite pair must have a similar fractional part, at least if they sufficiently overlap. Therefore, it is possible to estimate the SD–UPDs precisely and to apply them for a successful ambiguity fixing in PPP.

At the beginning of this paper, the SD-ambiguities between satellites within a global reference network are statistically analyzed to study the temporal and spatial behavior of SD-UPDs (Ge et al. 2006a). Based on their proven stability, an approach for their accurate and reliable estimation is established using data and solutions of a reference network. Afterwards, a strategy for fixing SD-ambiguities at a single station is presented which uses the estimated SD-UPD corrections to remove the fractional part of the SD-ambiguities. Finally, the fixing strategy is validated using about 450 stations of the International GNSS Service (IGS). The efficiency of the fixing strategy is confirmed by an improvement of about 27% in the repeatability and of 30% in the agreement with the IGS weekly solutions for the station east component.

2 Resolution of double-difference ambiguities

The resolution of DD-ambiguities is addressed now with concentration on PPP-related issues and on those formulas used in the following sections, including the concept of integer phase ambiguity and uncalibrated phase delays, as well as the ambiguity resolution using ionosphere-free observations. Although this is mostly common knowledge, such a detailed description is necessary for a more understandable presentation.

2.1 The uncalibrated phase delay

The basic model for the dual-frequency GPS carrier-phase and pseudo-range observations from receiver k to satellite i, in unit of length, is

$$L_{m}{}_{k}^{i} = -\lambda_m \phi_{m}{}_{k}^{i} = \varrho_k^i - \frac{\kappa}{f_m^2} + \lambda_m b_{m}{}_{k}^{i}, \tag{1}$$

$$P_{m}{}_{k}^{i} = \varrho_k^i + \frac{\kappa}{f_m^2}, \tag{2}$$

where $\phi_{m}{}_{k}^{i}$ and $P_{m}{}_{k}^{i}$ are carrier-phase and pseudo-range observations in frequency band m with corresponding wavelength λ_m and frequency f_m; $b_{m}{}_{k}^{i}$ is the phase ambiguity; ϱ_k^i is the non-dispersive delay, including geometric delay, tropospheric delay, clock biases and any other delay which affects all the observations identically; the second term on the right side is the ionospheric delay. The phase center correction and the phase windup effect (Wu et al. 1993) must be considered in modelling. The multipath effect and noise are not included for clarity. The receiver- and satellite-dependent pseudo-range biases (Schaer and Steigenberger 2006) are also ignored because the constant shifts have no substantial effect on the ambiguity fixing in the presented strategy as discussed in Sect. 3.

The carrier-phase ambiguity is composed of the following three terms:

$$b_{m}{}_{k}^{i} = n_{m}{}_{k}^{i} + \Delta\phi_m{}^{i} - \Delta\phi_{m\,k}, \tag{3}$$

where $n_{m}{}_{k}^{i}$ is the integer ambiguity; $\Delta\phi_m{}^{i}$ and $\Delta\phi_{m\,k}$ are UPDs in the receiver and in the satellite transmitter, respectively. The UPDs are not integer values, thus prevent the resolution of the integer ambiguities. However, they are identical for common instruments, are stable to better than a nanosecond (Blewitt 1989), and are eliminated while forming DD-ambiguities between two satellites i, j and two receivers k, l:

$$b_{m}{}_{k,l}^{i,j} = b_{m}{}_{k}^{i} - b_{m}{}_{k}^{j} - (b_{m}{}_{l}^{i} - b_{m}{}_{l}^{j}) = n_{m}{}_{k,l}^{i,j}, \tag{4}$$

where the super index pair i, j is for the single-difference between satellites i and j, while the sub index pair k, l is for the single-difference between receivers k and l.

2.2 Ionosphere-free solutions

For PPP, as well as for large scale networks, in order to eliminate the ionosphere effect, the well-known ionosphere-free observation is used:

$$L_{c_k}^{\,i} = \frac{f_1^2}{f_1^2 - f_2^2} L_{1_k}^{\,i} - \frac{f_2^2}{f_1^2 - f_2^2} L_{2_k}^{\,i} = \varrho_k^i + \lambda_1 b_{c_k}^{\,i}, \tag{5}$$

where $b_{c_k}^{\,i}$ is the related ambiguity and usually expressed as the following combination of wide- and narrow-lane for ambiguity fixing:

$$\begin{aligned} b_{c_k}^{\,i} &= \frac{f_1^2}{f_1^2 - f_2^2} b_{1_k}^{\,i} - \frac{f_1 f_2}{f_1^2 - f_2^2} b_{2_k}^{\,i} \\ &= \frac{f_1}{f_1 + f_2} b_{n_k}^{\,i} + \frac{f_1 f_2}{f_1^2 - f_2^2} b_{w_k}^{\,i}, \end{aligned} \tag{6}$$

where $b_{w_k}^{\,i}$ and $b_{n_k}^{\,i}$ are wide- and narrow-lane, respectively.

Denoting the epoch-dependent parameters, for example receiver and satellite clocks, with $\mathbf{u}$, the estimated ambiguity parameters with $\mathbf{b_c}$ and all the others with $\mathbf{x}$, the linear observation equations of Eq. (5) at epoch e with the weight matrix $\mathbf{P}_e$ reads:

$$\mathbf{v}_e = \mathbf{A}_e\mathbf{x} + \mathbf{B}_e\mathbf{b}_c + \mathbf{C}_e\mathbf{u}_e + \mathbf{l}_e, \quad \mathbf{P}_e. \tag{7}$$

Their contribution to the normal equation system after elimination of $\mathbf{u}_e$ is

$$\begin{bmatrix} \mathbf{A}_e^T\overline{\mathbf{P}}_e\mathbf{A}_e & \mathbf{A}_e^T\overline{\mathbf{P}}_e\mathbf{B}_e \\ & \mathbf{B}_e^T\overline{\mathbf{P}}_e\mathbf{B}_e \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{b_c} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_e^T\overline{\mathbf{P}}_e\mathbf{l}_e \\ \mathbf{B}_e^T\overline{\mathbf{P}}_e\mathbf{l}_e \end{bmatrix} \tag{8}$$

with

$$\overline{\mathbf{P}}_e = \mathbf{P}_e - \mathbf{P}_e\mathbf{C}_e(\mathbf{C}_e^T\mathbf{P}_e\mathbf{C}_e)^{-1}\mathbf{C}_e^T\mathbf{P}_e.$$

The final normal equation system after accumulating all the observations is

$$\begin{bmatrix} \mathbf{N_{xx}} & \mathbf{N_{xb}} \\ & \mathbf{N_{bb}} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{b_c} \end{bmatrix} = \begin{bmatrix} \mathbf{w_x} \\ \mathbf{w_b} \end{bmatrix}. \tag{9}$$

Instead of mapping ZD-ambiguities to DD-ambiguities, in this study, the ZD-ambiguities are estimated in order to select the "most-easy-to-fix" DD-ambiguities according to their post-fit estimates and variances (Ge et al. 2005). The corresponding solution reads

$$\begin{bmatrix} \mathbf{x} \\ \mathbf{b_c} \end{bmatrix} = \begin{bmatrix} \mathbf{Q_{xx}} & \mathbf{Q_{xb}} \\ & \mathbf{Q_{bb}} \end{bmatrix} \begin{bmatrix} \mathbf{w_x} \\ \mathbf{w_b} \end{bmatrix}, \tag{10}$$

with

$$\sigma_0^2 = \frac{\sum \mathbf{l}_e \overline{P}_e \mathbf{l}_e - \mathbf{x}^T\mathbf{w_x} - \mathbf{b_c}^T\mathbf{w_b}}{n - t},$$

where n and t are the number of observations and unknown parameters, respectively. Equation (10) is the real-valued solution normally used as basic information for ambiguity fixing.

2.3 Ambiguity fixing

From Eq. (6), the DD-ambiguity of satellites i and j and receivers k and l can be expressed as

$$b_{c_{k,l}}^{\,i,j} = \frac{f_1}{f_1 + f_2} b_{n_{k,l}}^{\,i,j} + \frac{f_1 f_2}{f_1^2 - f_2^2} b_{w_{k,l}}^{\,i,j}. \tag{11}$$

The wide- and narrow-lane ambiguities cannot be estimated and fixed simultaneously due to the rank deficiency of the normal equation system. Usually, the wide-lane is fixed using the corresponding carrier-phase and pseudo-range combination (Melbourne 1985; Wübbena 1985). After its successful fixing, the narrow-lane and its related standard deviation (STD) are derived from the real-valued solution and tested whether also fixable. An ionosphere-free ambiguity is fixed only when both its wide- and narrow-lane are fixed.

2.3.1 Wide-lane fixing

The wide-lane phase and range observations from satellite i to receiver k are defined as,

$$\begin{aligned} L_{w_k}^{\,i} &= \frac{f_1}{f_1 - f_2} L_{1_k}^{\,i} - \frac{f_2}{f_1 - f_2} L_{2_k}^{\,i} \\ &= \varrho_k^i - \frac{\kappa}{f_1 f_2} + \lambda_w b_{w_k}^{\,i}, \end{aligned} \tag{12}$$

$$P_{w_k}^{\,i} = \frac{f_1}{f_1 + f_2} P_{1_k}^{\,i} - \frac{f_2}{f_1 + f_2} P_{2_k}^{\,i} = \varrho_k^i + \frac{\kappa}{f_1 f_2}. \tag{13}$$

From Eqs. (12) and (13), one gets the following observation equation at each epoch with the wide-lane ambiguity as unknown:

$$b_{w_k}^{\,i} = \frac{(L_{w_k}^{\,i} - P_{w_k}^{\,i})}{\lambda_w}. \tag{14}$$

Then the solution reads

$$\hat{b}_{wk}^{\,i} = \langle b_{w_k}^{\,i} \rangle, \tag{15}$$

$$\sigma_{\hat{b}_{wk}^{\,i}} = \sqrt{\frac{\langle (b_{w_k}^{\,i} - \hat{b}_{wk}^{\,i})^2 \rangle}{N_k^i}}, \tag{16}$$

where $\langle\,\rangle$ denotes the average over epochs, N_k^i the number of the observations.

The estimate and its STD of a DD-ambiguity are calculated thereafter with

$$\hat{b}_{wk,l}^{\,i,j} = \hat{b}_{wk}^{\,i} - \hat{b}_{wk}^{\,j} - (\hat{b}_{wl}^{\,i} - \hat{b}_{wl}^{\,j}), \tag{17}$$

$$\sigma_{\hat{b}_{wk,l}^{\,i,j}} = \sqrt{\sigma^2_{\hat{b}_{wk}^{\,i}} + \sigma^2_{\hat{b}_{wk}^{\,j}} + \sigma^2_{\hat{b}_{wl}^{\,i}} + \sigma^2_{\hat{b}_{wl}^{\,j}}}. \tag{18}$$

The fixing decision is made according to the probability P_0 (fixation to the nearest integer), which is calculated, e.g., with the following formula (Dong and Bock 1989):

$$P_0 = 1 - \sum_{i=1}^{\infty} \left[\operatorname{erfc}\left(\frac{i-|b-n|}{\sqrt{2}\sigma}\right) - \operatorname{erfc}\left(\frac{i+|b-n|}{\sqrt{2}\sigma}\right) \right] \quad (19)$$

with

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-t^2} dt, \quad (20)$$

where b is the estimate and σ its STD, and n the nearest integer of b.

For a given confidence level α, e.g., 0.1% as usual, the ambiguity can be fixed to its nearest integer if P_0 is larger than $1-\alpha$, otherwise not.

2.3.2 Narrow-lane fixing

Only for an ambiguity with fixed wide-lane, its narrow-lane and the related STD can be derived according to Eq. (11) as

$$\hat{b}_{nk,l}^{\ i,j} = \frac{f_1 + f_2}{f_1} \hat{b}_{ck,l}^{\ i,j} - \frac{f_2}{f_1 + f_2} \hat{n}_{wk,l}^{\ i,j}, \quad (21)$$

$$\sigma_{\hat{b}_{nk,l}^{\ i,j}} = \frac{f_1 + f_2}{f_1} \sigma_{\hat{b}_{ck,l}^{\ i,j}}, \quad (22)$$

where $\hat{n}_{wk,l}^{\ i,j}$ is the fixed integer value of the wide-lane; $\hat{b}_{ck,l}^{\ i,j}$ and $\sigma_{\hat{b}_{ck,l}^{\ i,j}}$ are estimate and STD of the ionosphere-free ambiguity calculated based on the real-valued solution of Eq. (10). The fixing decision for the narrow-lane can be made in the same way as for the wide-lane using Eq. (19).

If both wide- and narrow-lane are fixed, the related integer ambiguity can be reconstructed with Eq. (11) and inserted as known into the normal equation system with DD-ambiguities (Dong and Bock 1989; Blewitt 1989), or as constraint imposed to the normal equation system with original ZD-ambiguities (Ge et al. 2005).

3 Resolution of ambiguities at a single station

In PPP, because only one single station is involved, only SD-ambiguities between satellites can be defined. For ionosphere-free observations, they can be derived from Eq. (6) in the form of integer ambiguities and UPDs by considering Eq. (3) as

$$\begin{aligned} b_{c_k}^{\ i,j} &= \frac{f_1}{f_1 + f_2} b_{n_k}^{\ i,j} + \frac{f_1 f_2}{f_1^2 - f_2^2} b_{w_k}^{\ i,j} \\ &= \frac{f_1}{f_1 + f_2} (n_{n_k}^{\ i,j} + \Delta\phi_n^{\ i,j}) \\ &\quad + \frac{f_1 f_2}{f_1^2 - f_2^2} (n_{w_k}^{\ i,j} + \Delta\phi_w^{\ i,j}). \end{aligned} \quad (23)$$

Each SD-UPD has an integer and a fractional part. For recovering the integer nature of SD-ambiguities, only the fractional part is critical, whereas the integer part is anyway not separable from the integer ambiguities.

For the wide-lane ambiguity, the estimate and its STD can be computed with the following formula, similar to Eqs. (17) and (18):

$$\hat{b}_{wk}^{\ i,j} = \hat{b}_{wk}^{\ i} - \hat{b}_{wk}^{\ j}, \qquad \sigma_{\hat{b}_{wk}^{\ i,j}} = \sqrt{\sigma^2_{\hat{b}_{wk}^{\ i}} + \sigma^2_{\hat{b}_{wk}^{\ j}}}. \quad (24)$$

If the SD–UPD $\Delta\phi_w^{\ i,j}$ or its fractional part $\delta\phi_w^{\ i,j}$ is known or can be estimated with an accuracy of $\sigma_{\delta\phi_w^{\ i,j}}$, the corrected wide-lane SD-ambiguity has a naturally integer value. Its estimate and STD are derived as

$$\tilde{n}_{wk}^{\ i,j} = \hat{b}_{wk}^{\ i,j} - \delta\phi_w^{\ i,j}, \quad \sigma_{\tilde{n}_{wk}^{\ i,j}} = \sqrt{\sigma^2_{\hat{b}_{wk}^{\ i,j}} + \sigma^2_{\delta\phi_w^{\ i,j}}}. \quad (25)$$

The fixing decision can be made using Eq. (19) as for DD-ambiguities. Therefore, the key issue for fixing SD-ambiguities is whether and how the SD–UPDs or their fractional parts can be estimated accurately. The same holds for fixing of narrow-lane ambiguities.

Instead of using Eq. (23) directly, the following reformulation is used for retrieving of narrow-lane ambiguities and reconstructing of fixed ionosphere-free ambiguities.

Assuming the wide-lane SD-ambiguity in Eq. (25) can be fixed to integer $\hat{n}_{wk}^{\ i,j}$, one gets the narrow-lane ambiguity with Eq. (23) as

$$\begin{aligned} &n_{n_k}^{\ i,j} + \Delta\phi_n^{\ i,j} + \frac{f_2}{f_1 - f_2}(n_{wk}^{\ i,j} - \hat{n}_{wk}^{\ i,j} + \Delta\phi_w^{\ i,j}) \\ &\quad = \frac{f_1 + f_2}{f_1} \hat{b}_{ck}^{\ i,j} - \frac{f_2}{f_1 - f_2} \hat{n}_{wk}^{\ i,j}. \end{aligned} \quad (26)$$

The difference between $n_{wk}^{\ i,j}$ and $\hat{n}_{wk}^{\ i,j}$ is caused mainly by the pseudo-range biases and the integer part of the $\Delta\phi_w^{\ i,j}$ which are both constant. Therefore, the second and third terms on the left side of Eq. (26) are constant for a satellite pair, and can therefore be merged into the UDP for the narrow-lane which is consistent with the used wide-lane UDP.

With the following definition

$$\Delta\overline{\phi}_n^{\ i,j} = \Delta\phi_n^{\ i,j} + \frac{f_2}{f_1 - f_2}(n_{wk}^{\ i,j} - \hat{n}_{wk}^{\ i,j} + \Delta\phi_w^{\ i,j}), \quad (27)$$

$$\hat{b}_{nk}^{\ i,j} = n_{n_k}^{\ i,j} + \Delta\overline{\phi}_n^{\ i,j}, \quad (28)$$

Eq. (26) can be written as

$$\hat{b}_{nk}^{\ i,j} = \frac{f_1 + f_2}{f_1} \hat{b}_{ck}^{\ i,j} - \frac{f_2}{f_1 - f_2} \hat{n}_{wk}^{\ i,j}. \quad (29)$$

Equation (29) is the same as Eq. (21) except that SD-ambiguities are used. Similar to Eq. (22), the related STD is derived as

$$\sigma_{\hat{b}_{nk}^{\ i,j}} = \frac{f_1 + f_2}{f_1} \sigma_{\hat{b}_{ck}^{\ i,j}}. \quad (30)$$

Thereafter, in the same way as for the wide-lane, the narrow-lane ambiguity can be fixed to integer if the fractional part of the narrow-lane UPD in Eq. (28), which is actually the mixture of the fractional parts of both wide- and narrow-lane, can be estimated precisely.

Through this reformulation, the estimated wide-lane UPD is only needed for making the fixing decision, but not for the deriving of narrow-lane ambiguities and reconstructing of the fixed ionosphere-free ambiguities. Thus, only biases in the estimated narrow-lane UPDs enter into the reconstructed fixed ambiguities and contaminate estimated parameters and have therefore to be estimated as precisely as possible.

One must be aware of that if the estimated wide-lane UPD is biased, which is inevitable because the integer parts are not separable with the ambiguities, the fixed values for all the ambiguities of the same satellite pair will be shifted by a common integer value [see Eq. (25)]. Such a common shift in wide-lane will result in a constant change in their narrow-lane ambiguities according to Eqs. (27)–(29), and can be absorbed by the narrow-lane UPDs. Therefore, for the ambiguity fixing, the knowledge of the fractional parts of the UPD corrections is sufficient. For the same reason, the constant pseudo-range biases in satellites have also no impact on the ambiguity fixing.

In summary, an ambiguity can be fixed on the SD-level for PPP in the similar way as on the DD-level, if the fractional parts of the SD-UPDs can be estimated and applied as corrections to SD-ambiguities.

4 Estimation of the uncalibrated phase delays

The stability of the SD–UPDs can be measured by the consistency of the fractional parts of all SD-ambiguities of the same satellite pair. Therefore, the spatial and temporal behavior of wide- and narrow-lane ambiguities are investigated by analyzing their estimates in PPP mode from about 180 IGS stations observed through the days 106 to 119 in 2006.

4.1 Behavior of UPD in wide-lane

To confirm that the wide-lane UPD for a satellite pair is a constant, the wide-lane estimates of all ZD-ambiguities and their STDs are calculated using Eqs. (12)–(16). Here, elevation-dependent weighting is also suggested, for example (Gendt et al. 2003),

$$p(E) = \begin{cases} 1.0, & \text{for } E \geq 30^\circ \\ 2\sin(E), & \text{otherwise} \end{cases} \tag{31}$$

Then, all possible SD-ambiguities between pairs of satellites are defined according to the start and end time of all the ZD-ambiguities at each station, if enough common observed data are available.

The number of possible wide-lane SD-ambiguities for a pair of satellites depends on the simultaneous observations of the two satellites and the carrier-phase discontinuities. The estimates of these SD-ambiguities and their STDs are calculated with Eq. (24) using the results of the corresponding ZD-ambiguities. In order to avoid possible biased estimates due to, for example, large multipath effects, SD-ambiguities with an observation time shorter than 20 minutes are ignored. For the same reason, those with a STD larger than 0.2 cycles are also rejected. Obviously, outliers have to be detected and removed as usual while taking the average.

The UPD estimate for satellite pair i, j is calculated by averaging the fractional parts of all related SD-ambiguities as

$$\delta\hat{\phi}_w{}^{i,j} = \langle \mathrm{frac}(\hat{b}_{wk}{}^{i,j}) \rangle, \tag{32}$$

$$\sigma^2_{\delta\hat{\phi}_w{}^{i,j}} = \frac{\langle [\mathrm{frac}(\hat{b}_{wk}{}^{i,j}) - \delta\hat{\phi}_w{}^{i,j}]^2 \rangle}{N^{i,j}}, \tag{33}$$

where $N^{i,j}$ is the number of all SD-ambiguities for this satellite pair, $frac()$ is a function to return the positive fractional part of the input variable. It must be pointed out that the fractional part for each individual SD-ambiguity from the function $frac()$ can fall into $(0.0, \epsilon)$ or $(1.0 - \epsilon, 1.0)$ where ϵ is about three times the STD of the fractional parts, if the true fractional part for the satellite pair is near to zero. Therefore, this special case must be taken into account in order to obtain a meaningful answer from Eqs. (32) and (33). Furthermore, the sign-constrained robust adjustment (Xu 2005) is also implemented to identify outliers. The same is done for the estimation of the narrow-lane fractional part.

The results reveal a rather variable quality of pseudo-range observations. Generally, the pseudo-range quality of receivers using the cross-correlation approach, for example, Trimble 4000 SE and Rogue SNR 8000, is poorer than that of the others. From the authors' experience, they should not be included in the UPD estimation. In the presented approach, wide-lane UPDs are estimated iteratively to enable automatic removal of stations with poor code pseudo-range quality.

The estimated UPDs are applied to all the wide-lane SD-ambiguities and the fixing decisions are made according to the fixing probability from Eq. (19) (see Sect. 5 for details). The fixing rate of the corrected ambiguities at each station is calculated. Stations with a fixing rate lower than 80% will not be used in the next iteration. From the test network, about 60 stations are excluded, mainly those with cross-correlation receivers. These excluded stations are referred to as **CC**-receiver later.

As an example, Fig. 1 gives the estimated wide-lane UPDs for day 106 in 2006. The solid dots show the estimated UPDs of all satellites referenced to PRN 30. Clearly, UPDs

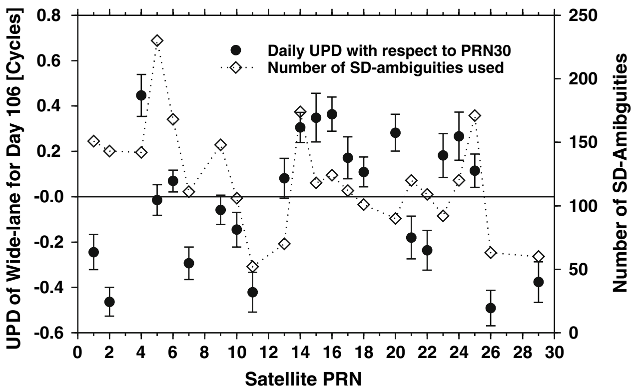

Fig. 1 Estimated wide-lane SD–UPDs for all satellites with respect to PRN30 for day 106, 2006. The *solid dots* are for the fractional part of the SD–UPDs, with the *error bars* for the STDs of the SD-ambiguities involved in the estimation; and the *triangle* the number of the SD-ambiguities

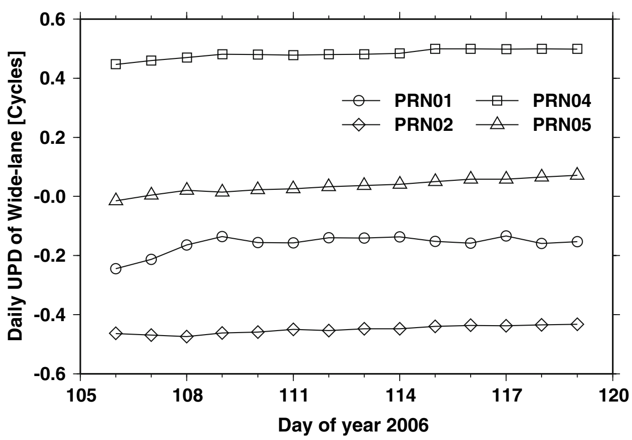

Fig. 2 Stability of the daily wide-lane SD–UPDs for satellites PRN01, PRN02 and PRN03 with respect to PRN30

are significantly non-zero. Without applying its correction, SD-ambiguities are usually far away from integer and thus cannot be fixed. The error bars show the STDs of the fractional parts of the SD-ambiguities, which is an indicator of the stability of the UPDs, usually smaller than 0.1 cycles and on average over all satellites 0.08 cycles. The triangles in Fig. 1 indicate the number of SD-ambiguities for each satellite pair used in the calculation. The estimated UPDs have a precision of 0.05 cycles, which is good enough for fixing the wide-lane ambiguities according to Eq. (19).

Moreover, as shown in Fig. 2, the estimates of the same satellite pair but for different days agree with each other better than 0.05 cycles in RMS. Therefore, wide-lane UPDs can even be predicted for real-time applications with an update time interval of several days.

Another very effective way to validate the UPD estimates is to check the fixing efficiency when applying these corrections. Under the round-off criterion of 0.25 cycles, about

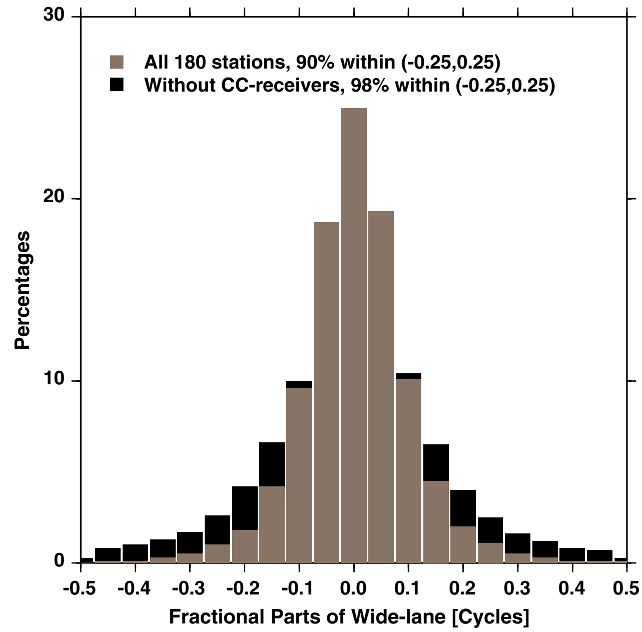

Fig. 3 Distribution of the fractional parts of all the UPD-corrected wide-lane SD-ambiguities. *Black* is the result from the network of all 180 stations, where about 90% are close to an integer within 0.25 cycles, whereas *gray* is for that of the network without CC-receivers, where about 98% of SD-ambiguities close to integer

90% and 98% of all defined wide-lane SD-ambiguities can be fixed for the network with and without CC-receivers, respectively; the distributions of the fractional parts for all the SD-ambiguities from both networks are shown in Fig. 3.

4.2 Behavior of UPD in narrow-lane

After the wide-lane is fixed, the related narrow-lane ambiguity and its STD are derived using Eqs. (29) and (30). As typical examples, for satellites PRN22, PRN23 and PRN24 with respect to PRN01, the fractional parts of the narrow-lane SD-ambiguities are presented in Fig. 4, arranged according to the epoch number at the middle of the corresponding data interval for the SD-ambiguity. In general, the fractional parts are not constant, but change in time and space. The change in time is much larger than that in space, which is indicated in Fig. 4 by the scatter along the y-axis.

For PRN22, the change reaches up to 0.4 cycles, while the two others are relative stable. From Eqs. (27)–(29), the fractional part of a narrow-lane ambiguity contains not only the UPD, but also the bias in the estimated ambiguity, which are contaminated by inaccurate modelling of the observations. This results in the fluctuation of the fractional parts. Thus, the time-dependent change has to be considered in order to provide high-quality UPDs, unlike to the use of daily means as corrections for the wide-lane. Additionally, the wavelength of the narrow-lane is about 10 cm, and therefore reliable fixing

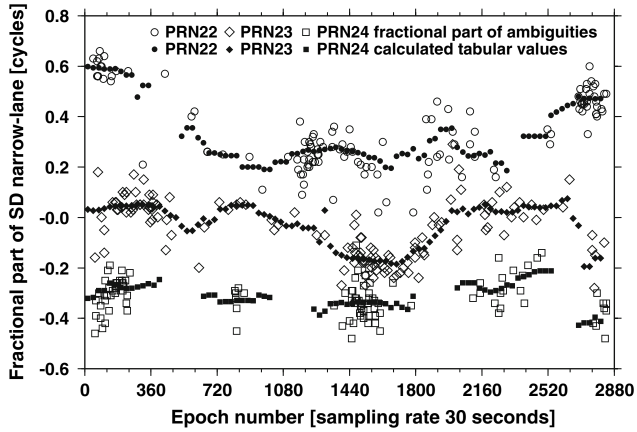

Fig. 4 Fractional parts of narrow-lane SD-ambiguities from the network without CC-receivers for the selected satellites with PRN01 as reference, shown with *open symbols*. In general, the fractional parts are not constant, but change up to 0.4 cycles with time. However, they are very stable over a certain time span and thus can be represented by piece-wise-constant functions. The *smaller solid symbols* denote the calculated tabular corrections with a step-size of 15 min

must be based on very accurate UPD estimates, if possible better than 1 cm RMS.

Fortunately, as shown in Fig. 4, the fractional parts of the narrow-lane ambiguties are rather stable over a certain time span and the averaged values over that interval can be used to correct the combined effect of the UPDs and the ambiguity biases. The cancellation of the common ambiguity biases is most likely causing the improvement in position accuracy.

As one possible approach, all those SD-ambiguities in the reference network, which are able to form DD-ambiguities with the narrow-lane SD-ambiguity to be fixed, could be used to compute its UPD corrections. That means, the chosen SD-ambiguities have to be for the same satellite pair and have to have enough common observing time with the ambiguity to be fixed. The disadvantage of this approach is that the fractional part of each SD-ambiguity in the reference network must be kept as information for the calculation of UPD, which is inconvenient especially for real-time applications.

A more efficient approach is to characterize the time-dependent change of UPDs using a piece-wise-constant function with a certain step-size for each satellite pair. In practise, they can be represented by a table of the UPD corrections. Each tabular value is calculated by averaging the fractional parts of all those SD-ambiguities, which covers at least half of its validity time interval:

$$\delta\hat{\phi}_n{}^{i,j}(t_m, t_{m+1}) = \langle \mathrm{frac}(\hat{b}_{nk}{}^{i,j}), \quad \mathbf{if}(t_s, t_e) \cap (t_m, t_{m+1}) > 0.5\Delta t \rangle, \tag{34}$$

where (t_m, t_{m+1}) is the validity time interval of the m-th tabular value, Δt step-size, (t_s, t_e) the validity interval of the SD-ambiguities of the satellite pair.

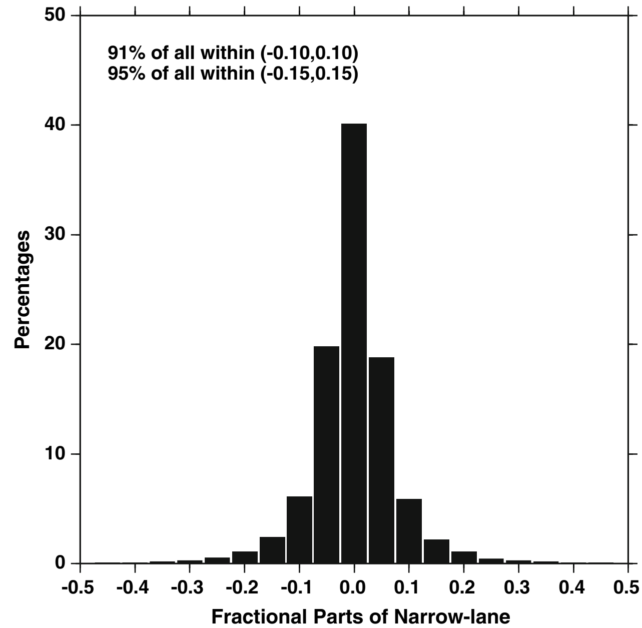

Fig. 5 Distribution of the fractional parts of all the narrow-lane SD-ambiguities with fixed wide-lane from the network without the CC-receivers after having applied the estimated UPD corrections. More than 91% are closer to an integer within 0.1 cycles, and 95% within 0.15 cycles

For ambiguity fixing of daily data, after careful testing, a 15-min step-size is used. A maximum of 96 points are needed for each satellite pair, only. For the satellite pairs selected as examples, the calculated values are shown in Fig. 4 with the corresponding solid symbols.

For a to-be-fixed ambiguity, the correction is the average of all those tabular values, which have an overlap longer than half the step-size with its valid interval:

$$\delta\hat{\phi}_n{}^{i,j}(t_s, t_e) = \langle \delta\hat{\phi}_n{}^{i,j}(t_m, t_{m+1}), \quad \mathbf{if}\ (t_s, t_e) \cap (t_m, t_{m+1}) > 0.5\Delta t \rangle. \tag{35}$$

Figure 5 shows the distribution of the fractional parts of all narrow-lane SD-ambiguities with fixed wide-lane after applying narrow-lane UPD corrections. The corrections are computed as a mean value of those UPDs with at least 10 minutes common observing time. More than 91% are closer to an integer within 0.1 cycles and can normally be fixed reliably to the related integer according to Eq. (19). The fixing rate is about 1–2% lower if the corrections are calculated from the 15-min tabular data.

4.3 Summary of UPD estimation

From the performed data analysis, the fractional parts of SD–UPDs for both wide- and narrow-lane are significantly non-zero. Thus, without proper corrections, ambiguities within PPP can never be fixed to integers. The fractional part of SD–UPDs for wide-lane are rather stable in time, and can be estimated directly from code-range and carrier-phase observations with a precision better than 0.05 cycles from rather dense networks, for example, a global network of about 100 stations. The accuracy of the estimates only depends on the accuracy of the observations and the number of contributing stations. However, inconsistent pseudo-range biases, such as biases in CC-receivers, which cannot be eliminated by forming double-differences, will contaminate the results. The same holds for strong multipath environments. Therefore, data from receivers of poor quality and at lower elevations should be down-weighted. Applying the estimated UPDs, more than 98% of the wide-lane SD-ambiguities can be fixed to integers.

The ambiguity parameters are more or less biased by the possible inaccurate modelling that appears over short time intervals as systematic biases. It leads to the fact that the fractional parts of narrow-lane ambiguities change with time. However, over a certain time interval of about 2–3 h, they are very stable and agree with others very well, especially if they have enough common observations. Therefore, time-dependent UPD corrections for narrow-lane ambiguities are much more reliable than those of the daily average and lead to a much higher ambiguity fixing rate.

5 Ambiguity fixing for PPP based on estimated UPD

The studies will concentrate on daily solutions for precise applications in geodynamics and geodesy, where the data interval is long enough so that ambiguities are usually well estimated and the fixing decision can simply be made according to the estimates and STDs of the ambiguities (Blewitt 1989; Dong and Bock 1989; Ge et al. 2005).

Products needed by PPP users, i.e., orbits, clocks and Earth rotation parameters, can be obtained from the IGS or generated by analyzing a reference network in a similar way. For ambiguity-fixing, wide- and narrow-lane UPDs have to be estimated. The first ones are estimated for each satellite pair as a constant for one day directly from pseudo-range and carrier-phase observations. Therefore, they are independent from the analysis model. The second ones are represented by a set of tabular correction values in order to consider their time-dependant change due to the existence of modelling errors as demonstrated in the above investigation.

Narrow-lane UPD tables can be generated based on the ionosphere-free ambiguities of either network or PPP solutions. A network solution can be performed by users who generate orbit and clock products, for example, IGS Analysis Centers. In the PPP case, solutions have to be performed with IGS products for a set of reference stations and the narrow-lane ambiguities are derived based on the ionosphere-free ambiguities of the PPP solutions. Obviously, in this case the estimated UPDs fit best to the products of the applied PPP software, so that any impact from inconsistent software packages can be reduced significantly. Therefore, the latter approach is recommended and used in the following experimental test.

For a single station, PPP is performed with ionosphere-free observations and repeated until no new cycle slips or bad observations are detected anymore. Then all possible SD-ambiguities are defined, and their wide-lane ambiguities are estimated. The fixing decisions are made for the ambiguities corrected with the wide-lane UPDs. Afterwards, the following steps are carried out:

- From all SD-ambiguities with fixed wide-lane, the narrow-lane ambiguities and their STD are calculated based on the real-valued solution and the integer wide-lane values.
- For each narrow-lane ambiguity, its UPD correction is computed from the tabular UPD corrections, and the fixing decision is made in the same way as for the wide-lane.
- From all fixable ambiguities, an independent set is selected and fixed to the values constructed from the integer and the narrow-lane UPD values by applying constraints on the related ZD-ambiguities (Ge et al. 2005).

With the updated solution, all not-yet-fixed candidates are checked for fixing and corresponding constraints are added for newly fixed and independent ambiguities. This procedure is repeated until no more ambiguities can be fixed.

6 Validation

In order to validate the ambiguity fixing strategy for PPP, and to estimate its impact on the results, the UPD estimation approach and the ambiguity fixing strategy are implemented into the PANDA (Positioning And Navigation system Data Analyst) software package (Liu and Ge 2003). The software package has been developed at the GNSS Research and Engineering Center of Wuhan University, China, and has similar capabilities as those running at IGS Analyis Centers, but not involved in the generation of IGS products.

In principle, all the stations can be included in the UPD estimation. However, we simulate a system providing a PPP service, which generates necessary information from a reference network and transmits it to the user stations where PPP with ambiguity fixing is performed.

Data from about 450 IGS stations observed during days 106 to 119 in 2006 are used in the experimental validation. About 180 stations processed regularly by most of the Analysis Centers are chosen as a reference network because of their good distribution and performance. The others are for PPP test and are referred as user stations later. Again,

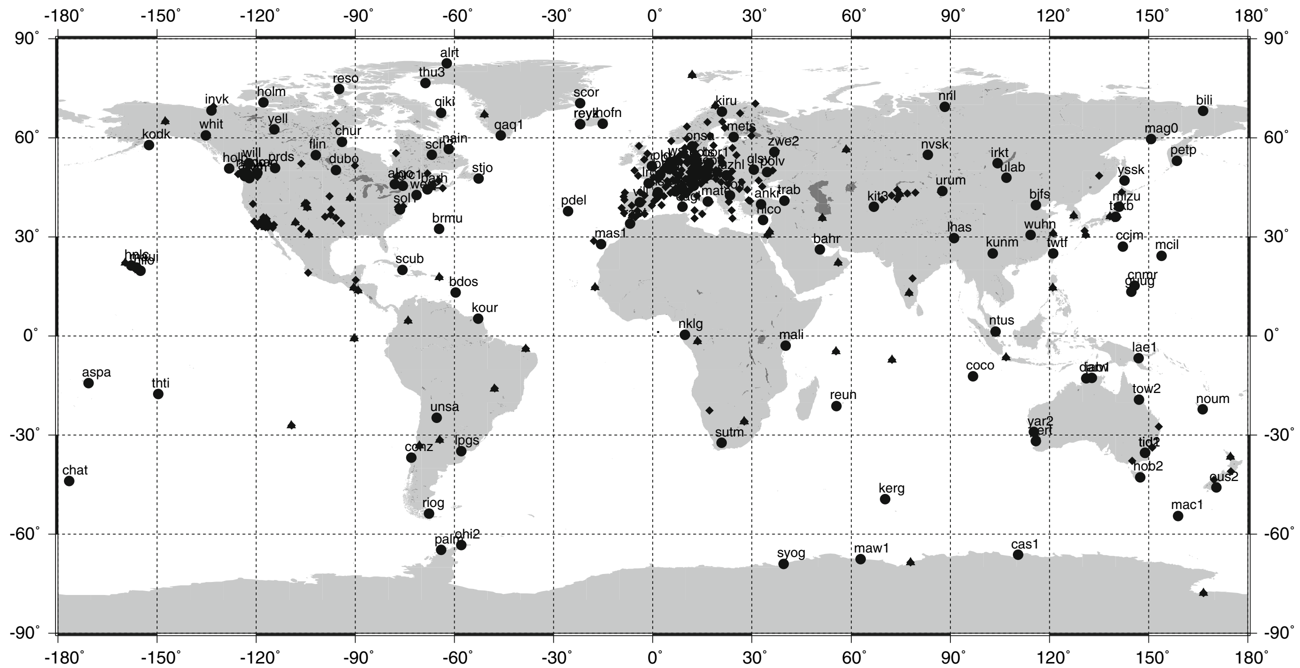

Fig. 6 Station distribution. About 180 stations are chosen as reference stations. Among them, 60 stations, marked with small triangles, are automatically removed from UPD estimation because of poor wide-lane quality. The large dots mark stations finally used in the estimation of UPDs. The small diamonds indicate the 270 user stations

as already mentioned in Sect. 4.1, about 60 of the selected reference stations are rejected from the estimation of UPDs because of poor code pseudo-range quality. Finally, there are about 120 reference stations and 330 user stations for each day (Fig. 6).

With the IGS Final orbits, clocks and Earth rotation parameters fixed, the reference stations are analyzed in PPP mode. Carrier-phase and code pseudo-range data are used with elevation-dependent weighting of Eq. (31). Receiver clocks are estimated epoch by epoch, zenith tropospheric delays are estimated every 60 minutes with an initial STD of 0.2 m and a power density of $20\,\mathrm{mm}/\sqrt{\mathrm{h}}$ (Gendt et al. 2003), station coordinates are estimated with an initial constraint of 0.2 m. ZD-ambiguities are constrained to the values calculated by code pseudo-ranges with an a-priori STD of about 20 m. The IGS relative antenna model (Mader 1999) is adopted to obtain results comparable with the corresponding IGS weekly solutions.

From the PPP solutions of the reference stations, wide-lane and narrow-lane UPDs are computed. The quality of the estimates was already presented in Sect. 3 while investigating their behavior and discussing estimation approaches.

The remaining 330 stations are processed in PPP mode using the same approach as for the reference stations, except the constraints of the station coordinates are relaxed to 1 m as usual for unknown stations. The ambiguity fixing is performed based on the real-valued PPP solutions and the estimated UPDs. For comparison with the IGS weekly solutions, ambiguity fixing is also done for the reference stations to obtain real-valued and fixed solutions for all the 450 stations in the PPP mode.

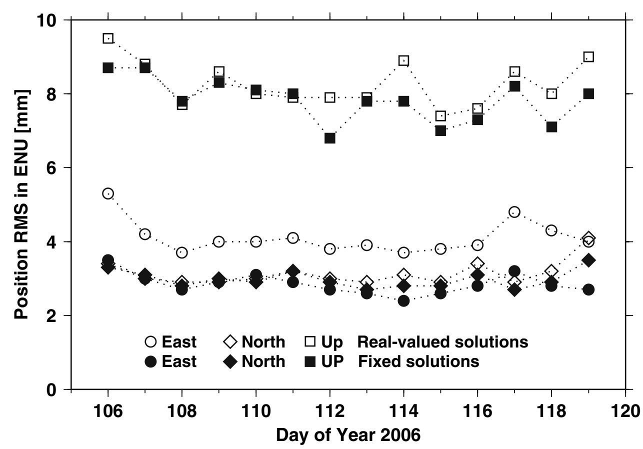

Fig. 7 RMS in the east, north and up directions for real-valued and fixed solutions compared with IGS weekly solutions; on average 4.1, 3.1, 8.3 mm for real-valued solution and 2.8, 3.0, 7.8 mm for fixed solution. The improvement in the east component is about 30%

In order to judge the efficiency of the ambiguity fixing strategy, PPP results are compared with the IGS weekly solutions using 7-parameter Helmert transformations between about 240 common stations. Figure 7 shows the RMS of the transformed residuals of the east, north and up components for both real-valued and fixed solutions. For the real-valued

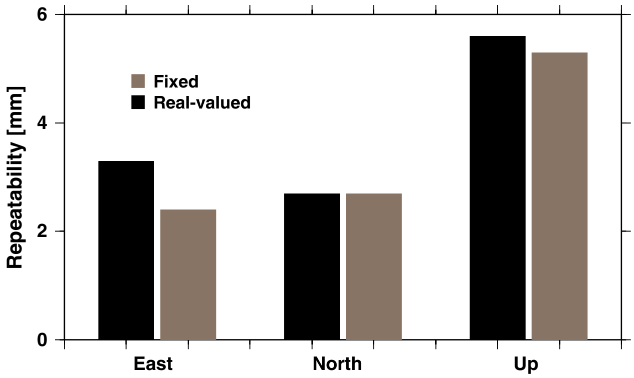

Fig. 8 Repeatability of station position in east, north and up directions for the real-valued and the fixed solutions. The east-component is improved by about 27% by ambiguity-fixing

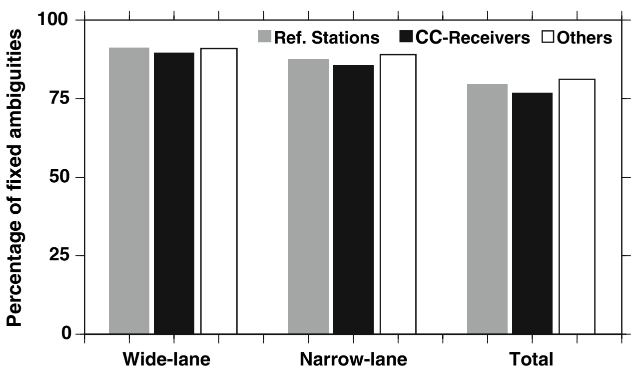

Fig. 9 Averaged fixing rate for stations in the reference network, for those with CC-receivers, and for the user stations. The fixing rate for wide-lane is the number of fixed wide-lane over the number of all independent SD-ambiguities. For narrow-lane it is the number of the fixed narrow-lane over the number of fixed wide-lane. The total fixing rate is the number of fixed narrow-lane over the number of the independent SD-ambiguities

solution the RMS are 4.1, 3.1, 8.3 mm in east, north and up, respectively. The larger RMS for east in the real-valued solutions is reduced from 4.1 to 2.8 mm by ambiguity fixing in PPP, and is now even marginally smaller than that for north, while the other two components are improved very slightly. This is comparable with the improvement in the network mode after applying ambiguity fixing.

The averaged repeatabilities for the real-valued solutions, shown in Fig. 8, are 3.3, 2.7 and 5.6 mm in east, north and up and 2.4, 2.7 and 5.3 mm for the fixed solutions. The repeatabilities are improved by about 27% by ambiguity fixing for the east component. Compared with the corresponding RMS of the Helmert transformation, the much better repeatabilities imply that PPP solutions have somehow station-specific biases, which should be investigated further.

Figure 9 gives the fixing rate for the reference stations, for those with CC-receivers and for the rest of the IGS stations. Be aware that the fixing rate is defined as the percentage of fixed ambiguities among the independent ones. Because the ambiguities are selected according to their fixing probabilities, there are no significant differences among the three types of stations: the fixing rate for CC-receivers is only slightly lower than the others. The fixing rate of the user stations is even higher than that of the reference stations, most likely because the user stations are mainly located in Europe and North America, where the dense coverage of IGS reference stations provide better UPDs.

From Fig. 9, about 80% of the independent ambiguities can be fixed. About half of the no-fixing is due to the wide-lane problem from the 90% fixing rate of wide-lane, while another half is caused by narrow-lane problem as only 90% of the ambiguities with fixable wide-lane can be fixed in narrow-lane.

7 Conclusions

It is demonstrated that UPDs, which originate at satellites and prevent the integer ambiguity resolution at the SD level, are rather stable. An approach is implemented using data from a dense reference network to estimate the fractional parts of the wide-lane and narrow-lane UPDs with high accuracy and reliability. They are applied to correct the SD-ambiguities from PPP solution and to fix them sequentially to integer as usually done for the DD-ambiguities in the network mode.

With data collected at about 450 IGS stations during days 106 to 119 in 2006, PPP using IGS Final products with and without ambiguity fixing were performed. Both the repeatabilities and the RMS of station coordinates compared to IGS weekly solutions show that an improvement of 30% in the east component was obtained by the ambiguity fixing, and the quality of the east component is even slightly better than that of the north.

The strategy has solved for the first time the PPP ambiguity-fixing problem on a single station basis. It is expected to be applied to kinematic PPP positioning in post-mission mode.

Acknowledgments All implementations in the PANDA software package necessary for this work was carried out under the framework of the "Earth Observing Data Spatial Information-Transform Mechanism of Geodetic Knowledge" project (No:2006CB701301) funded by Program 973 of China. We are grateful to two anonymous reviewers and Professor Will Featherstone for their critical comments and helpful suggestions, which improved the manuscript greatly.

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